Unbinned Maximum Likelihood Estimation of Polarization Angle Rotation in GX 13+1

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Introduction

GX 13+1 is a neutron star low-mass X-ray binary (NS-LMXB) **observed by *IXPE* three times in the soft state**.

Each observation has yielded distinct polarization properties including a **polarization angle rotation of ~70°/2 days in the first observation**. (Bobrikova+ 24a)

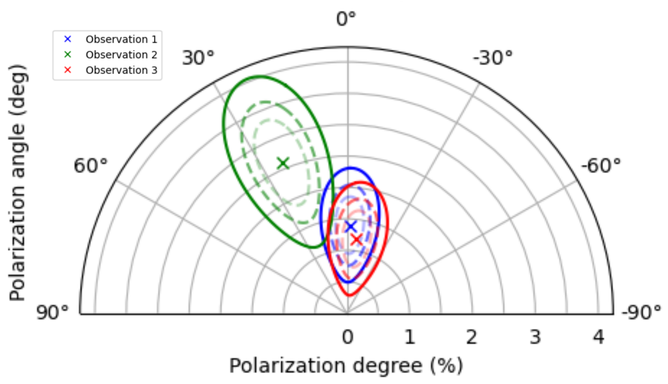

Figure 1. Overview of IXPE GX 13+1 observations (PCUBE analysis)

Polarimetry Methods

There are two main methods of calculating Stokes parameters:

PCUBE (Baldini+ 22, Kislat+ 15):

$$\psi_i \xrightarrow{q_i = cos2\psi_i,\ u_i = sin2\psi_i} q_i, u_i \xrightarrow{Q = \sum_{i=1}^{N} q_i \ \ U = \sum_{i=1}^{N} u_i} Q, U$$

Unbinned Maximum Likelihood Estimation:
(Marshall 21a, 21b, 23)

$$S(f_0, Q, U) = -2Nln(f_0) - 2\sum_i \ln(1 + Q\mu_i cos2\psi_i + U\mu_i sin2\psi_i)$$

$$\psi_i \xrightarrow{+2Kf_0 + 2K_\mu f_0 Q \int_0^{2\pi} w(\psi) cos2\psi d\psi + 2K_\mu f_0 U \int_0^{2\pi} w(\psi) sin2\psi d\psi} Q, U$$

Why Use Maximum Likelihood Estimation?

- Allows for testing model dependencies on multiple parameters such as polarization angle (PA) rotation rate
- Help refine our Stokes Q,U estimates by accounting for PA rotation
- Probes time-dependent and energy-dependent phenomena without need for binning
- Can be utilized in combination with a Bayesian method to provide physically-motivated results

Maximum Likelihood Results

Maximum likelihood results (red) agree with PCUBE results (blue) for all three IXPE observations of GX 13+1. (Bobrikova+ 24a, Bobrikova+ 24b)

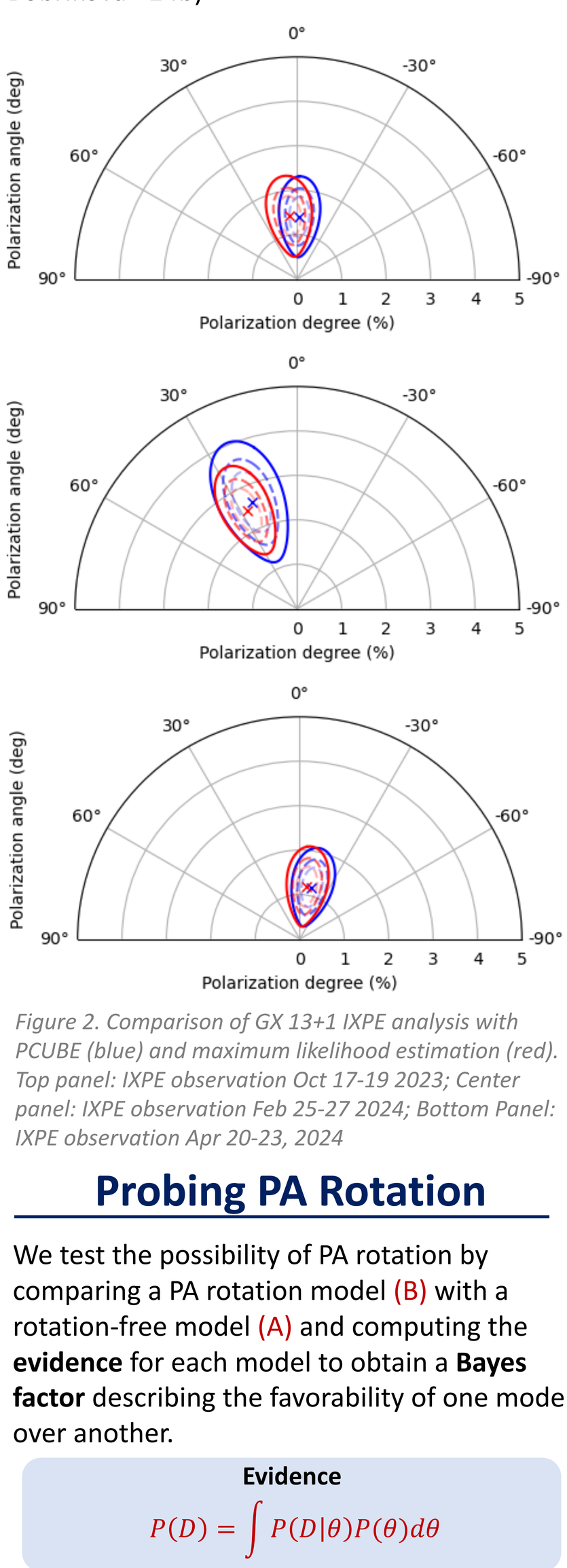

Figure 2. Comparison of GX 13+1 IXPE analysis with PCUBE (blue) and maximum likelihood estimation (red). Top panel: IXPE observation Oct 17-19 2023; Center panel: IXPE observation Feb 25-27 2024; Bottom Panel: IXPE observation Apr 20-23, 2024

Probing PA Rotation

We test the possibility of PA rotation by comparing a PA rotation model (B) with a rotation-free model (A) and computing the **evidence** for each model to obtain a **Bayes factor** describing the favorability of one model over another.

Evidence

$$P(D) = \int P(D|\theta)P(\theta)d\theta$$

Bayes Factor

$$B_{AB} = \frac{P(D|B)}{P(D|A)}$$

PA Rotation Results

PCUBE analysis shows significant rotation of **~70°/2 days** for the first IXPE observation of GX 13+1, with no significant rotation seen in the second or third IXPE observations (Bobrikova+ 24a, Bobrikova+ 24b).

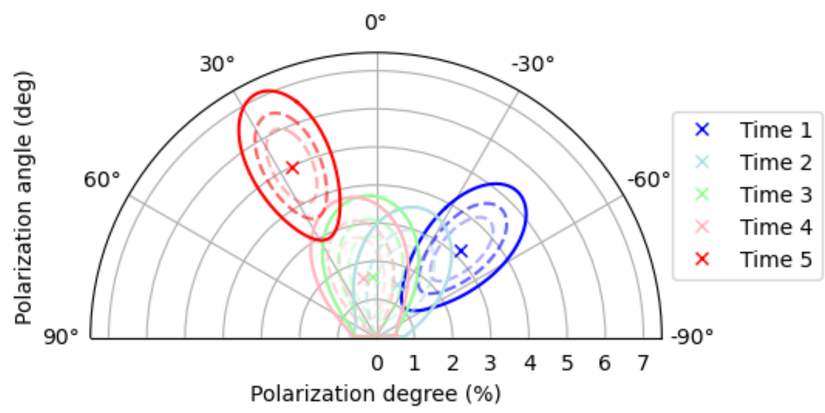

Figure 3. IXPE observation 02006801 of GX 13+1, segmented into 5 equal time bins of 10.5 hours each, illustrating PA rotation

The maximum likelihood method corroborates this**, finding a $41.2^{+5.2}_{-4.3}$ °/day rotation** for the first IXPE observation of GX 13+1 and no significant rotation seen in subsequent observations.

The PA rotation model produces a **significantly different PD of $2.58^{+0.28}_{-0.31}$%** versus $1.42^{+0.32}_{-0.24}$% in the rotation-free model

Physical Interpretations

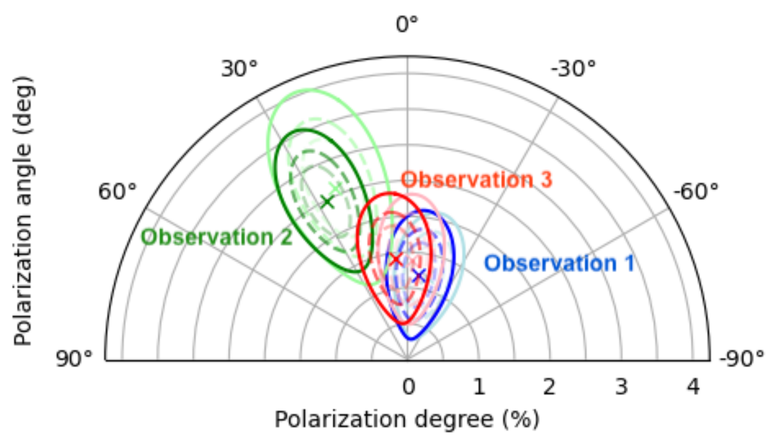

Figure 4. Summary of 3 previous IXPE observations of GX 13+1. Saturated colors represent maximum likelihood results and desaturated colors represent PCUBE results

PA rotation in GX 13+1 may be due to **accretion disk winds** or **misalignment of spin axis with orbital axis.** Further orbital phase-resolved observations can elucidate a preferred model.

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